King Abdulaziz University College of Science

DEPARTMENT OF MATHEMATICS

First Exam-Spring 2009-2010 14/4/2010 29/4/1431

1:20 minuts

Name:	
Student	ID:
Section:	

Course: Math 342

Title: Abstract Algebra I.

Instructor: Dr. Rola Hijazi

1. This exam consists of three parts

Part 1 True/False question (5 marks)	
Part 2 Fill in the blanks question (3 marks)	
part 3 Theory questions (12 marks)	
Total marks (20 marks)	

- 2. Answer **ALL** questions on the question sheets.
- 3. This exam sheet consists of 4 pages including this page.

I. True/ False

1	1 Every group has atmost one identity element.	
2	Finite nonempty subset of a group that is closed is a subgroup.	
3	In an abelian group $(ab)^{-1} = a^{-1}b^{-1}$.	
4	The additive group Z_n is a subgroup of the additive group Z .	
5	(Z_4, \cdot) is a group under multiplication.	
6	If g is a group element and $g^n = e$, then $ g = n$.	
7	Every element of a group generates a cyclic subgroup of that group.	
8	Every cyclic group has at least two generators.	
9	If $ a = n$ and $a^k = e$, then $n k$.	
10	If a group has an element of order 15 it must have at least 8 elements of order 15.	
11	For every positive integer n there exisits a cyclic group of order n.	

II. Fill in the blanks.

- (a) The elements of U(12) are
- (b) In U(12), $|U(12)| = \dots$ and $|5| = \dots$
- (c) Let $4 \in Z_{12}$, then $|4| = \dots$
- (d) Let $4 \in \mathbb{Z}$, then $|4| = \dots$
- (e) Let $5 \in \mathbb{Z}_{24}$, then the inverse of 5 is

- III. Consider the group $(Z_{20}, +)$. Solve the following questions.
 - (a) Find the generators of Z_{20} .
 - (b) Find all subgroups of Z_{20} .
 - (c) Determined the subgroup lattice of Z_{20} .
 - (d) List all generators for the subgroup of order 5.
 - (e) deduce all elements of order 5.

IV. Let G be a group, $H = \{a \in G : aHa^{-1} = H\}$. Show that $H \leq G$.

V. Show that $(Z_n, +)$ is a commutative group.