# King Abdulaziz University College of Science 

## Department of Mathematics

First Exam-Spring 2009-2010
14/4/2010
29/4/1431
1:20 minuts

Name: $\qquad$

## Student ID:

$\qquad$
Section: $\qquad$

Course: Math 342
Title: Abstract Algebra I.

Instructor: Dr. Rola Hijazi

1. This exam consists of three parts

| Part 1 True/False question (5 marks) |  |
| :--- | :--- |
| Part 2 Fill in the blanks question $(3$ marks) |  |
| part 3 Theory questions (12 marks) |  |
| Total marks (20 marks) |  |

2. Answer ALL questions on the question sheets.
3. This exam sheet consists of 4 pages including this page.

## I. True/ False

| 1 | Every group has atmost one identity element. |  |
| :--- | :--- | :--- |
| 2 | Finite nonempty subset of a group that is closed is a subgroup. |  |
| 3 | In an abelian group $(a b)^{-1}=a^{-1} b^{-1}$. |  |
| 4 | The additive group $Z_{n}$ is a subgroup of the additive group $Z$. |  |
| 5 | $\left(Z_{4}, \cdot\right)$ is a group under multiplication. |  |
| 6 | If $g$ is a group element and $g^{n}=e$, then $\|g\|=n$. |  |
| 7 | Every element of a group generates a cyclic subgroup of that <br> group. |  |
| 8 | Every cyclic group has at least two generators. |  |
| 9 | If $\|a\|=n$ and $a^{k}=e$, then $n \mid k$. |  |
| 10 | If a group has an element of order 15 it must have at least 8 <br> elements of order 15. |  |
| 11 | For every positive integer $n$ there exisits a cyclic group of <br> order n. |  |

## II. Fill in the blanks.

(a) The elements of $U(12)$ are $\qquad$
(b) $\operatorname{In} U(12),|U(12)|=$ $\qquad$ and $|5|=$
(c) Let $4 \in Z_{12}$, then $|4|=$ $\qquad$
(d) Let $4 \in Z$, then $|4|=$ $\qquad$
(e) Let $5 \in Z_{24}$, then the inverse of 5 is $\qquad$
III. Consider the group $\left(Z_{20},+\right)$. Solve the following questions.
(a) Find the generators of $Z_{20}$.
(b) Find all subgroups of $Z_{20}$.
(c) Determined the subgroup lattice of $Z_{20}$.
(d) List all generators for the subgroup of order 5.
(e) deduce all elements of order 5 .
IV. Let $G$ be a group, $H=\left\{a \in G: a H a^{-1}=H\right\}$. Show that $H \leq G$.
V. Show that $\left(Z_{n},+\right)$ is a commutative group.

