## KING ABDULAZIZ UNIVERSITY COLLEGE OF SCIENCE

## DEPARTMENT OF MATHEMATICS

## Fall 2011-2012

## Project 1, Math 342 Dr. Jehan A. Al-bar

Read "symmetries of a square" in chapter one of Contemporary Abstract algebra's book by J. Gallian.

Answer the following questions and show all details of your work.

- 1. Construct the dihedral group  $D_4$  and exhibit its cayley table.
- 2. What is the identity of  $D_4$ ?
- 3. Is  $D_4$  Abelian group?
- 4. List the members of  $H = \{x^2 : x \in D_4\}$  and  $K = \{x \in D_4 : x^2 = e\}$ .
- 5. Find the center of  $D_4$ .
- 6. Find the centralizer of the following elements of  $D_4$ ;  $C(R_{90}), C(H)$ , and C(D).
- 7. Determine the order of  $D_4$ .
- 8. Determine the order of each element in  $D_4$ .
- 9. Is  $D_4$  a cyclic group?
- 10. Express  $D_4$  in terms of permutation of the locations of each of the four corners of a square. In this setting notice that  $D_4$  is a subgroup of  $S_4$ .
- 11. Determine  $Inn(D_4)$ .

Hint: first you find the complete list of all inner automorphisms in  $D_4$ , then reduce your list by grouping the similar automorphisms together and then show that your reduced list consists of distinct automorphisms.

- 12. If  $\phi$  is an automorphism on  $D_4$  and  $\phi(R_{90}) = R_{270}$  and  $\phi(V) = V$ , what are  $\phi(D)$  and  $\phi(H)$ ?
- 13. Find the left regular representation of  $D_4$ .
- 14. Let  $K = \{R_0, R_{180}\}$ . Construct the factor group  $D_4/K$  and exhibit its multiplication table.
- 15. Consider a mapping  $\phi$  on  $D_4$  given by;  $\phi(R_0) = \phi(R_{180}) = R_0$ ,  $\phi(R_{90}) = \phi(R_{270}) = H$   $\phi(H) = \phi(V) = R_{180}$ ,  $\phi(D) = \phi(D') = V$ . Find Ker $\phi$ . Show that  $D_4/Ker\phi \simeq \phi(D_4)$ .