# King Abdulaziz University College of Science 

## Department of Mathematics

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Project 1, Math 342
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Read "symmetries of a square" in chapter one of Contemporary Abstract algebra's book by J. Gallian.
Answer the following questions and show all details of your work.

1. Construct the dihedral group $D_{4}$ and exhibit its cayley table.
2. What is the identity of $D_{4}$ ?
3. Is $D_{4}$ Abelian group?
4. List the members of $H=\left\{x^{2}: x \in D_{4}\right\}$ and $K=\left\{x \in D_{4}: x^{2}=e\right\}$.
5. Find the center of $D_{4}$.
6. Find the centralizer of the following elements of $D_{4} ; C\left(R_{90}\right), C(H)$, and $C(D)$.
7. Determine the order of $D_{4}$.
8. Determine the order of each element in $D_{4}$.
9. Is $D_{4}$ a cyclic group?
10. Express $D_{4}$ in terms of permutation of the locations of each of the four corners of a square.
In this setting notice that $D_{4}$ is a subgroup of $S_{4}$.
11. Determine $\operatorname{Inn}\left(D_{4}\right)$.

Hint: first you find the complete list of all inner automorphisms in $D_{4}$, then reduce your list by grouping the similar automorphisms together and then show that your reduced list consists of distinct automorphisms.
12. If $\phi$ is an automorphism on $D_{4}$ and $\phi\left(R_{90}\right)=R_{270}$ and $\phi(V)=V$, what are $\phi(D)$ and $\phi(H)$ ?
13. Find the left regular representation of $D_{4}$.
14. Let $K=\left\{R_{0}, R_{180}\right\}$. Construct the factor group $D_{4} / K$ and exhibit its multiplication table.
15. Consider a mapping $\phi$ on $D_{4}$ given by; $\phi\left(R_{0}\right)=\phi\left(R_{180}\right)=R_{0} \quad, \quad \phi\left(R_{90}\right)=\phi\left(R_{270}\right)=H$ $\phi(H)=\phi(V)=R_{180} \quad, \quad \phi(D)=\phi\left(D^{\prime}\right)=V$.
Find Ker $\phi$. Show that $D_{4} / \operatorname{Ker} \phi \simeq \phi\left(D_{4}\right)$.

