King Abdulaziz University
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Question sheet 2, Math 342

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Try to do as much as you can of the following questions:

1. Let G be the group of rotations of a plane about a point P in the plane. Thinking of G as a group of permutations of the plane, describe the orbit of a point Q in the plane.

2. Let α and $\beta \in S_n$.

   (a) Prove that $\alpha\beta$ is even if and only if α and β are both even or both odd.

   (b) Prove that if α is even then α^{-1} is even and if α is odd then α^{-1} is odd.

   (c) Prove that ϵ, the identity permutation, is an even permutation.

   (d) Prove that A_n, the subset of even permutations is a subgroup of S_n.

3. Give an example to show that a group can be isomorphic to a proper subgroup of itself.

4. Suppose that $\phi : Z_{50} \to Z_{15}$ is a group homomorphism with $\phi(7) = 6$.

   (a) Determine $\phi(x)$.

   (b) Determine the image of ϕ.

   (c) Determine the kernel of ϕ.

   (d) Determine $\phi^{-1}(3)$. That is determine the set of all elements that map to 3.
   Hint: Assume that $\phi(1) = k$.

5. Suppose that ϕ is a homomorphism from $U(30)$ to $U(30)$ and that $Ker\phi = \{1, 11\}$. If $\phi(7) = 7$ find all elements of $U(30)$ that map to 7

6. Suppose that ϕ is a homomorphism from a finite group G onto $\overline{G}$ and that $\overline{G}$ has an element of order 8.
   Prove that G has an element of order 8.