King Abdulaziz University
College of Science

Department of Mathematics

Second Exam-Fall 2010-2011
5/1/2011
30/1/1432

1:20 minuts

Name: ....

Student ID: ....

Section: ....

Course: Math 342

Title: Abstract Algebra I.

Instructor: Dr. Jehan A. Al-bar

1. This exam consists of three parts

Part 1 True/False question (5 marks)	
Part 2 Fill in the blanks question (5 marks)	
part 3 Theory questions (10 marks)	
Total marks (20 marks)	

2. Answer **ALL** questions on the question sheets.
3. This exam sheet consists of **4 pages** including this page.

I. **True/ False**

1	For $\alpha = (123)$ and $\beta = (45)$, $|\alpha\beta| = 6$.	
2	A function $f : \mathcal{R} \to \mathcal{R}$ defined by $f(x) = x^2$ is a permutation.	
3	Every group is isomorphic to a group of permutation.	
4	If α and β are disjoint cycles then $\alpha\beta = \beta\alpha$.	
5	Every cyclic group of order n is isomorphic to Z_n.	
6	The mapping on $\mathcal{R}$ under addition given by $\phi(x) = x^3$ is an isomorphism.	
7	$U(10) \simeq Z_4$.	
8	If G is finite group and $a \in G$, then $|a| \mid |G|$.	
9	If G is a group and H is a subgroup of G, then the left coset aH is a subgroup of G.	
10	If $aH = Ha$, then $ah = ha$ for all $h \in H$.	

II. **Fill in the blanks.**

(a) The order of (156432) is ...., and the order of (134)(257896) is ....

(b) The permutation (15632) is an .... permutation, while the permutation (2734) is an .... one.

(c) If

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 1 & 5 & 2 & 7 & 4 & 6 \end{bmatrix}, \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 1 & 2 & 4 & 3 & 5 & 7 \end{bmatrix}$$

Then α^{-1} =...

and $\alpha\beta$ =...

(d) $|S_4|$ =......................., $|A_4|$ =...

(e) For a finite cyclic group G of order 8, G is isomorphic to ....

(f) In the group $(Z_8, +)$, the left cosets of the subgroup $< 4 >= \{\overline{0}, \overline{4}\}$ are ....
....

III. Let G be a group and $|G| = 8$, show that G must have an element of order 2.

IV. Let G be a finite cyclic group, $|G| = n$. Prove that $G \simeq Z_n$.

V. Let G be a group and H be a subgroup of G. For $a, b \in G$, prove that $aH = bH$ or $aH \cap bH = \emptyset$.

VI. Show that the converse of Lagrange theorem it not true.
Hint: use the group

$$A_4 = \{e, (12)(34), (13)(24), (14)(23), (123), (234), (134), (124), (132), (324), (314), (214)\}$$