King Abdulaziz University
College of Science

Department of Mathematics

Final Exam-Spring 2009-2010
12/6/2010 - 29/6/1431
2 hours

Name: ....

Student ID: ....

Section: ....

Course: Math 342

Title: Abstract Algebra I.

Instructors: Dr. Jehan A. Al-bar & Dr. Rola A. Hijazi.

1. This exam consists of three parts

Part 1 True/False question (5 marks)	
part 2 Theory questions (... marks)	
I (3 marks) II (5 marks) III (3 marks)	
IV (...marks) VI (3 marks) V (7 marks)	
Part 3 Applications (12 marks)	
I (2 marks) II (1.5 marks) III (3 marks)	
IV (2 marks) VI (1.5 marks) V (2 marks)	
Total marks (40 marks)	

2. Answer **ALL** questions on the question sheets.
3. This exam sheet consists of **7 pages** including this page.

True/ False

1	Every subgroup of an Abelian group is normal.	
2	The left coset aH is a subgroup of G.	
3	All generators of Z_{20} are prime numbers.	
4	If $H \leq G$ and $a \in G$, then $aH = Ha$.	
5	A factor group of an Abelian group is Abelian.	
6	Every function is a permutation if and only if the function is one-one.	
7	Every cyclic group is Abelian.	
8	If $\phi : G \to \overline{G}$ is a group homomorphism and $e, \overline{e}$ are the identities in G and $\overline{G}$ respectively, then $\phi(e) = \overline{e}$.	
9	S_{10} has 10 elements.	
10	H is a normal subgroup of a group G if and only if $xHx^{-1} \subseteq H$ for all $x \in G$.	

Theory Questions

I. Consider the group of nonzero rational numbers $G = Q - \{0\}$ with multiplication $(G, *)$. For $H = \{2^n, n \in Z\}$ show that H is a subgroup of G.

II. Let G be a group and H be a normal subgroup of G, show that the set $G/H = \{aH : a \in G\}$ with the multiplication is defined by $(aH)(bH) = abH$ is a group. Moreover, if G is Abelian group, show that G/H is Abelian.

III. Consider the groups $(R^+, *)$ and $(R, +)$ and define the mapping

$$\alpha : R^+ \to R \qquad \text{by} \qquad \alpha(a) = log_{10}a.$$

Show that α is an isomorphism.

IV. Let G be a group and for any $g \in G$ define $T_g : G \to G$ by

$$T_g(x) = gx \qquad \text{for all} \qquad x \in G.$$

If T_g is a permutation on the set of elements of G and

$$\overline{G} = \{T_g : g \in G\}$$

is the group of permutations under the composition $T_g T_h = T_{gh}$, prove that the mapping $\phi : G \to \overline{G}$ defined by $\phi(g) = T_g$ is an isomorphism.

V. Using Lagrange Theorem prove that a group of prime order is cyclic.

VI. Consider the groups G and $\overline{G}$ and let $\phi : G \to \overline{G}$ be a group homomorphism. Show that:

(a) $Ker\phi$ is a normal subgroup of G,

(b) $G/Ker\phi \approx \phi(G)$.

Applications

I. Construct Cayley table for $U(12)$.

II. List all the subgroups of Z_{20}.

III. Find the order of the following elements:

(a) In S_5, $|(345)(245)| =$............

(b) In $(Z_{20}, +)$, $|4| =$.............

(c) In $Z_{24}/ < 8 >$, $|14+ < 8 > | =$............

IV. Show that $U(8)$ is isomorphic to $U(12)$.

V. Let G be a group of order 60. What are the possible orders for the subgroups of G?

VI. Consider the group of integers with addition $(Z, +)$. For the subgroup $H = \{0, \pm 6, \pm 12, \pm 18, \pm 24, ...\}$ of Z find all the left cosets of H in Z.